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1 Poroelastic effects destabilize mildly rate-strengthening friction to  
2 generate stable slow slip pulses

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7 **Abstract**

Slow slip events on tectonic faults, sliding instabilities that never accelerate to inertially limited ruptures or earthquakes, are one of the most enigmatic phenomena in frictional sliding. While observations of slow slip events continue to mount, a plausible mechanism that permits instability while simultaneously limiting slip speed remains elusive. Rate-and-state friction has been successful in describing most aspects of rock friction, faulting, and earthquakes; current explanations of slow slip events appeal to rate-weakening friction to induce instabilities, which are then stalled by additional stabilizing processes like dilatancy or a transition to rate-strengthening friction at high slip rates. However, the temperatures and/or clay-rich compositions at slow slip locations are almost ubiquitously associated with rate-strengthening friction. In this study, we propose a fundamentally different instability mechanism that may reconcile this contradiction, demonstrating how slow slip events can nucleate with mildly rate-strengthening friction. We identify two destabilizing mechanisms, both reducing frictional shear strength through reductions in effective normal stress, that counteract the stabilizing effects of rate-strengthening friction. The instability develops into slow slip pulses. We quantify parameter controls on pulse length, propagation speed, and other characteristics, and demonstrate broad consistency with observations of tectonic slow slip events as well as laboratory tribology experiments.

8 *Keywords:* friction, stability and bifurcation, geological material, porous material, slip  
9 pulses

## 1. Introduction

Frictional instabilities are intrinsically linked with shear fracturing and material failure[1]. Earthquakes are notable examples of such instabilities, featuring explosive, inertially limited rupture growth on faults following gradual development of instability. Yet not all faults slip in earthquakes; some slide steadily in response to tectonic loading. This diversity in sliding behavior is well explained by rate-and-state friction, an experimentally based description of how the friction coefficient,  $f$ , evolves with sliding velocity (i.e., slip rate,  $V$ ) and sliding history. The fault shear strength  $\tau = f(\sigma - p)$ , the product of friction coefficient  $f$  and effective normal stress, the difference between compressive total stress  $\sigma$  and pore fluid pressure  $p$ . It is widely thought[2] that instabilities during sliding require rate-weakening friction, in which  $f$  decreases with increasing  $V$  (following a transient rate-strengthening response that stabilizes short-wavelength perturbations). Likewise, rate-strengthening friction is linked to aseismic slip, which is thought to occur steadily in the absence of changes in loading.

Slow slip events are challenging to reconcile with this understanding. Slow slip occurs in subduction zones and possibly at the base of some strike-slip faults and is one component of a class of sliding events that includes low-frequency earthquakes, tectonic tremor, tsunami earthquakes, some landslides, and even stick-slip cycles on ice streams[3, 4]. Slow slip is also thought to play an important role in injection-induced seismicity and reservoir stimulation by hydraulic fracturing[5, 6]. In addition, slow slip has been observed in friction experiments, in particular experiments on hydrogels that report spontaneous nucleation of slip pulses that propagate faster than the loading speed but much slower than elastic wave speeds [7, 8, 9, 10, 11].

The challenge posed by slow slip events is to simultaneously explain their unstable nature (i.e., why the interface does not slide steadily) and why they do not continue to grow into ruptures. Current theories posit that slow slip events nucleate under rate-weakening friction, just like earthquakes, but then stall for a variety of reasons. These include a transition from rate-weakening to rate-strengthening friction with increasing  $V$ , stabilization by dilatancy,

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37 and interaction with frictional heterogeneities[12, 13, 14, 15, 16]. However, these theories  
38 are inconsistent with indications that slow slip occurs in regions with rate-strengthening  
39 friction, based on temperature conditions and/or clay-rich compositions[17, 18, 19, 20].

40 In this paper, we show that slow slip arises naturally from instabilities with mildly  
41 rate-strengthening friction. These instabilities arise from two distinct mechanisms, both  
42 involving configurations where slip couples to changes in effective normal stress  $\sigma - p$ . The  
43 first mechanism occurs during sliding between poroelastic solids, for which compression  
44 or dilation of material causes changes in pore pressure  $p$  that alter frictional strength  $\tau$   
45 on the sliding interface [21, 22, 23] The second mechanism arises during sliding at the  
46 interface between two dissimilar elastic solids, a process that alters total normal stress  $\sigma$   
47 on the interface and hence frictional strength  $\tau$  [24]. We focus primarily on the poroelastic  
48 mechanism, but point out a correspondence between the undrained poroelastic problem  
49 and the elastic bimaterial problem (at least for a linearized friction law). We identify and  
50 characterize the sliding instabilities through 1.) linear stability analysis of perturbations  
51 about steady sliding, and 2.) numerical simulations of nucleation and propagation of slip  
52 pulses with fully nonlinear rate-and-state friction. Both the stability analysis and numerical  
53 simulations are done for linear poroelastic solids.

54 This paper has four main sections. In Section 2 we present the conceptual and mathemat-  
55 ical model. In Section 3 we develop solutions and discuss results for linearized rate-and-state  
56 friction on a slip surface in a poroelastic medium. Specifically, we derive a characteristic  
57 equation that describes the stability of the slip surface to a Fourier mode perturbation. Sec-  
58 tion 4 describes numerical simulations that account for fully nonlinear frictional response.  
59 Finally, Section 5 discusses and interprets the results in the context of observations of slow  
60 slip in nature and laboratory experiments; furthermore, we speculate on the manifestation  
61 of the rate-strengthening instability in 3D with possible application to subduction zone slow  
62 slip events and other geological phenomena.

## 63 2. Model

64 In this section we elaborate on the conceptual, mathematical, and physical foundations  
65 of the model before presenting the linearized stability analysis and numerical simulations  
66 that will follow.

### 67 2.1. Fault structure, poroelastic effects, and coupling to fault strength

68 The mechanical and physical properties of the fault core, a region of the fault zone of high  
69 strain where slip localizes (Fig. 1a-b), are important to understanding how slip nucleates  
70 and propagates on natural faults. Fault cores are mechanically and chemically different  
71 from the surrounding damage zone and, beyond that, the intact host rock. The fault core  
72 is thin compared to many seismologically relevant length scales, having a thickness ( $w$  in  
73 Fig. 1b) ranging from a few centimeters to meters [25]. Fault cores in well-developed fault  
74 zones often have very low permeability [26, 27, 28] compared to the surrounding damage  
75 zone and host rock [29, 30]. It is worth noting that in the analysis of this paper, we  
76 will use the concept of a fault core to describe a thin layer, in which slip localizes, with  
77 different mechanical and hydrological properties than the surrounding medium. However,  
78 in nongeological applications, the fault core may be regarded as a tribofilm that is produced  
79 by long-term wear of the frictional interface.

80 In this study, we assume that slip has localized at the boundary of the fault core (Fig. 1a-  
81 b), a configuration that maximizes the potential for instability compared to other locations  
82 within the fault core, as shown subsequently. Field exposures of formerly active faults also  
83 often feature localization on one side of the core [31, 32, 33].

84 Spatially nonuniform slip in this configuration compresses material on one side of the  
85 interface and dilates it on the opposite side, altering fluid pressure through poroelastic  
86 coupling [21, 22, 23, 34, 35]. Furthermore, this process is asymmetric, with pore pressure  
87 increases on one end of a slipping zone matched by corresponding decreases in pore pressure  
88 on the opposite end. The relatively impermeable fault core delays pressure equilibration by  
89 flow across the core. These changes in pressure cause changes in the effective normal stress  
90 on the slip surface that asymmetrically alter the shear strength of the fault [23, 34, 35].

91 Slip localization to the boundary of the core thus gives rise to an asymmetry that favors  
 92 propagation in one direction, with the favorable direction determined by which boundary of  
 93 the core hosts the slip surface. A related bimaterial effect altering normal stress occurs during  
 94 sliding between elastically dissimilar solids [24, 36, 37, 38]. The strong sense of directionality  
 95 in both the poroelastic and elastic bimaterial problems is a characteristic property of slip  
 96 pulses (Fig. 1c). Experiments confirm that slip instabilities on poroelastic and/or elastic  
 97 bimaterial interfaces often develop into propagating slip pulses [9, 39]. Coupling of slip  
 98 to normal stress is not unique to poroelastic and bimaterial interfaces. Interfaces of elastic  
 99 materials with identical mechanical properties also produce such coupling if the materials  
 100 lack geometric reflection symmetry[40].

101 In this study, we consider sliding on a slip surface at the edge of a fault core of width  
 102  $w$  with mobility (permeability divided by fluid viscosity)  $\kappa_c$ . The rock outside the fault  
 103 core has a possibly different mobility  $\kappa$ . For simplicity, we do not distinguish between the  
 104 damage zone and host rock in this study.

## 105 *2.2. Linear poroelasticity*

106 Here we describe the governing equations of quasi-static linear isotropic poroelasticity  
 107 and the interface conditions that we impose in our problem. The displacements  $u_i$  and pres-  
 108 sure changes  $p$  are governed by a set of four coupled partial differential equations. Assuming  
 109 that body forces are negligible, these are [e.g. 41]

$$Gu_{i,kk} + \frac{G}{1-2\nu}u_{k,ki} = \alpha p_{,i} \quad (1)$$

110 and

$$\frac{1}{M}p_{,t} - \kappa p_{,kk} = -\alpha u_{k,kt}, \quad (2)$$

111 where summation over repeated indices is implied and subscript  $,t$  denotes the partial time  
 112 derivative and  $,k$  denotes the partial spatial derivative in direction  $x_k$ . The material param-  
 113 eters are the shear modulus  $G$ , drained Poisson ratio  $\nu$ , mobility  $\kappa$ , Biot-Willis coefficient  $\alpha$ ,  
 114 and Biot modulus  $M$ . From Eq. 1 we see that the scaled pore pressure gradient,  $\alpha p_{,i}$ , acts

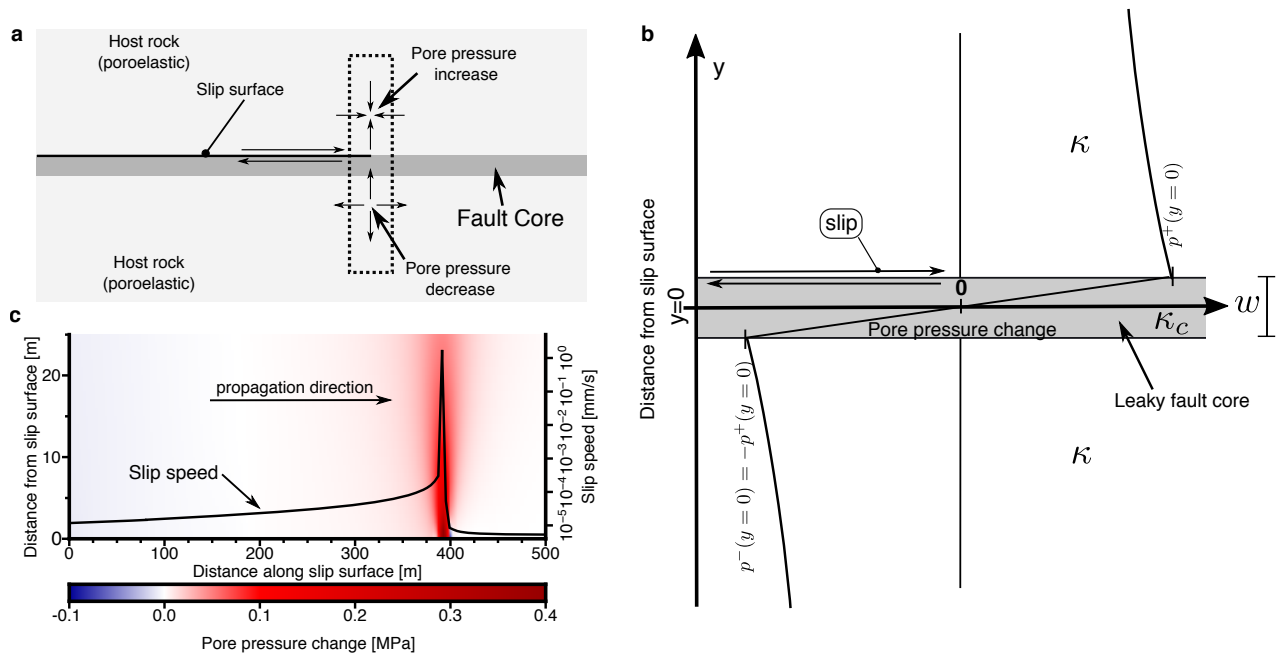


Figure 1: **a** Schematic of fault zone with slip localized on one side of the low permeability core, idealized in our study as having infinitesimal width with respect to the perturbation wavelength  $\lambda$ . Pressure changes are caused by compression/dilation of the near-fault material. **b** Zoomed-in view of dashed box in **a**, showing pore pressure change (with respect to a positive initial value) across the sliding interface. For across-fault diffusion we consider the core to have thickness  $w \ll \lambda$  and mobility  $\kappa_c$ , which may be different from the mobility  $\kappa$  of the surrounding rock. **c** Simulated slow slip pulse (black line with scale on the right axis) from Section 4; elevated pore pressure around the slip front weakens the interface, facilitating propagation.

115 as an effective body force. In Eq. 2, the scaled dilation rate,  $-\alpha M u_{k,kt}$ , is a source term  
 116 to a diffusion equation for pore pressure. The Biot-Willis parameter  $\alpha$  (between 0 and 1)  
 117 establishes the two-way coupling between the pore fluid phase and the elastic solid matrix.  
 118 We refer to Detournay and Cheng [41] for more detailed discussion of various poroelastic  
 119 parameters. In other sections, different sets of five parameters will be introduced if they  
 120 provide simpler expressions. Specifically, we use Skempton's coefficient  $B$ , the undrained  
 121 Poisson ratio  $\nu_u$ , and the hydraulic diffusivity  $c = \kappa M$ . Note that  $B$  relates undrained  
 122 response in pore pressure  $p$  perturbations to changes in mean stress:  $p = -B \Delta \sigma_{kk} / 3$  [22]  
 123 and is always between 0 and 1. We may relate  $B$  and  $\nu_u$  to the material parameters in Eqs.  
 124 1 and 2 using the following equations:

$$B = \frac{3M\alpha(1-2\nu)}{2G(1+\nu) + 3M\alpha^2(1-2\nu)}, \quad (3)$$

$$\nu_u = \frac{2G\nu + M\alpha^2(1-2\nu)}{2G + 2M\alpha^2(1-2\nu)}; \quad (4)$$

125 furthermore, a relationship between  $B$  and  $\nu_u$  is given by

$$B = \frac{3(\nu_u - \nu)}{\alpha(1-2\nu)(1+\nu_u)}. \quad (5)$$

126 In this study we seek a solution of Eqs. 1 and 2 for two half-spaces under the assumption  
 127 of 2-D plane strain deformation, thus reducing the system to three coupled partial differential  
 128 equations. We utilize an  $x$ - $y$  Cartesian coordinate system with  $y = 0$  being the sliding  
 129 interface. Our first objective is to obtain linear relations between slip and stress and pressure  
 130 change on the interface, which are used in subsequent sections when enforcing a specific  
 131 interface friction law. To obtain these linear relations, the following boundary and interface  
 132 conditions are imposed:



$$\lim_{y \rightarrow 0^\pm} u_x^+ - u_x^- = \delta, \quad (6)$$

$$\lim_{y \rightarrow 0^\pm} u_y^+ - u_y^- = 0, \quad (7)$$

$$\lim_{y \rightarrow \pm\infty} u_i^\pm = 0, \quad (8)$$

$$\lim_{y \rightarrow \pm\infty} p^\pm = 0, \quad (9)$$

$$\lim_{y \rightarrow 0^\pm} \sigma_{xy}^+ - \sigma_{xy}^- = 0, \quad (10)$$

$$\lim_{y \rightarrow 0^\pm} \sigma_{yy}^+ - \sigma_{yy}^- = 0, \quad (11)$$

133 where superscripts + or - represent the upper ( $y > 0$ ) or lower ( $y < 0$ ) half-spaces. The  
 134 first equation imposes slip  $\delta$  across the interface. The second equation assures that no  
 135 opening or interpenetration occurs on the interface. The third requires displacements and  
 136 stresses to vanish at infinity; the fourth requires pore pressure changes and fluid flux to  
 137 also vanish at infinity. The fifth and sixth equations enforce Newton's third law across the  
 138 interface. Two more conditions are required on the fault to fully specify the problem, which  
 139 describe the pore pressure or flux conditions on the interface. We formulate these conditions  
 140 in Section 2.3. These boundary conditions are formulated to characterize perturbations  
 141 around steady sliding and thus do not include far-field loading or prestress associated  
 142 with compression of the medium and shear resistance to spatially uniform steady sliding.  
 143 Furthermore, solutions to the linear problem stated above can be utilized together with a  
 144 friction law that prescribes some relation between shear stress, normal stress, pore pressure,  
 145 slip, slip rate, etc. Specifically, in Section 2.5 we combine these solutions with a linearized  
 146 rate-and-state friction law.

### 147 *2.3. Leaky fault model*

148 We next introduce two pressure and fluid flow interface conditions on the slip surface. We  
 149 assume that slip perturbations have wavelengths  $\lambda$  much larger than the fault core thickness  
 150  $w$  (Figure 1b). This scale separation, together with symmetry (or antisymmetry) of fields

151 across the fault, permits application of an approximate leaky fault model recently introduced  
 152 by Song and Rudnicki[42]. The leaky fault model accounts for flow across the fault core via  
 153 linear relations between pore pressure and its gradient in the fault-normal direction on the  
 154 two sides of the slip surface:

$$\left. \frac{dp^\pm}{dy} \right|_{y=0^\pm} = \pm \frac{\kappa_c}{\kappa} \frac{2p^\pm}{w}. \quad (12)$$

155 We note that if  $\kappa_c/\kappa \rightarrow 0$  then  $dp^\pm/dy \rightarrow 0$  at  $y = 0^\pm$ , thus providing boundary conditions  
 156 corresponding to an impermeable fault core. However, if  $\kappa/\kappa_c \rightarrow 0$  then  $p^\pm \rightarrow 0$ , which  
 157 corresponds to a fully permeable fault core.

#### 158 2.4. Solution of poroelastic problem with imposed slip

159 The governing equations and interface conditions (with imposed slip) can be solved  
 160 analytically in the Fourier-Laplace domain. We carry out a Fourier transform in fault-  
 161 parallel distance  $x$  and Laplace transform in time  $t$ ; this joint transform is defined as

$$\hat{\delta}(s, k) = \int_0^\infty \int_{-\infty}^\infty \delta(t, x) e^{-ikx-st} dx dt \quad (13)$$

162 for slip and similarly for other fields. The procedure in Appendix A provides linear relations  
 163 between the transformed shear stress change on the interface,  $\hat{\tau}$ , pore pressure change on  
 164 the two sides of the interface,  $\hat{p}^\pm$ , and slip,  $\hat{\delta}$ :

$$\hat{\tau} = -\frac{G|k|\hat{\delta}}{2(1-\nu_u)} H_1(s, k) \quad (14)$$

165 and

$$\hat{p}^\pm = \mp \frac{ikGB\hat{\delta}}{3} \frac{1+\nu_u}{1-\nu_u} H_2(s, k), \quad (15)$$

166 where

$$H_1(s, k) = 1 - \frac{2(\nu_u - \nu) ck^2}{1 - \nu} \frac{1 + \mathcal{F}}{s \mathcal{F} + \sqrt{1 + s/ck^2}} \left( \sqrt{1 + s/ck^2} - 1 \right), \quad (16)$$

167 and

$$H_2(s, k) = \frac{\sqrt{1 + s/ck^2} - 1}{\sqrt{1 + s/ck^2} + \mathcal{F}}, \quad (17)$$

168 in which  $\mathcal{F}$  is a nondimensional parameter (given a fixed  $k$ ) that characterizes the importance  
 169 of flux across the fault:

$$\mathcal{F} = \frac{\kappa_c}{\kappa} \frac{2}{|k|w}. \quad (18)$$

170 There is no change in total normal stress on the interface,  $\hat{\sigma} = 0$ .

171 Note that both  $H_1(s, k)$  and  $H_2(s, k)$  go to unity in the limit where  $ck^2/s \rightarrow 0$  assuming  
 172  $\mathcal{F} < \infty$ . We will refer to this as the undrained limit, where the change in effective normal  
 173 stress is the largest. We explore this limit later in detail due to mathematical simplicity and  
 174 the physically interesting effects that arise from changes in the effective normal stress. If  
 175  $\mathcal{F} \rightarrow \infty$ , then  $\hat{p}^\pm \rightarrow 0$ , which corresponds to a fully permeable fault, in which case there is  
 176 no change in effective normal stress on the fault.

### 177 2.5. Rate-and-state friction

178 Frictional sliding on the slip surface is governed by rate-and-state friction, which provides  
 179 a relation between shear strength  $\tau$ , effective normal stress  $\sigma' = \sigma - p$ , and friction coefficient  
 180  $f$  that depends on sliding velocity  $V$  and state variable  $\Psi$ . The latter obeys a state evolution  
 181 equation.

182 In the first part of this study, we perform a linear stability analysis using a general form  
 183 of linearized rate-and-state friction, valid for small perturbations about a steady sliding  
 184 solution at slip speed  $V_0$ , that encompasses a broad class of steady state friction and state  
 185 solution laws[36]:

$$\frac{d\tau}{dt} = \frac{a\sigma'_0}{V_0} \frac{dV}{dt} + (f_0 - \alpha_{LD}) \frac{d\sigma'}{dt} - \frac{V_0}{L} \left[ \tau - f_0\sigma' - \frac{(a-b)\sigma'_0}{V_0}(V - V_0) \right], \quad (19)$$

186 in which  $f_0$  is the steady-state coefficient of friction at sliding velocity  $V_0$ ,  $\alpha_{LD}$  is the Linker-  
 187 Dieterich constant [43],  $L$  is the state evolution distance,  $\sigma'_0 = \sigma_0 - p_0$  is the initial effective  
 188 normal stress, and  $a$  and  $b$  are dimensionless parameters that are related to the rate and  
 189 state dependence of friction, respectively.

190 In the second part of this study, we perform simulations with nonlinear rate-and-state  
 191 friction. For this we set  $\alpha_{LD} = 0$  and use a standard logarithmic dependence of steady state  
 192 friction coefficient on slip velocity together with the slip evolution law [e.g. 36]

$$f(V, \Psi) = a \operatorname{arcsinh} \left( \frac{V}{2V_0} e^{\Psi/a} \right), \quad (20)$$

$$\frac{\partial \Psi}{\partial t} = -\frac{V}{L} [f - f_{ss}(V)], \quad (21)$$

193 where  $\sigma'$  is the effective normal stress, the steady state friction coefficient is

$$f_{ss}(V) = f_0 + (a - b) \ln(V/V_0). \quad (22)$$

194 Friction is said to be rate-strengthening (under steady sliding conditions) if  $a - b > 0$  and  
 195 rate-weakening if  $a - b < 0$ . Linearization of Eqs. 20–22 yields Eq. 19.

### 196 2.6. Parameter values

197 For the analysis and simulations we present, we assume a set of reference parameters  
 198 (Table 1), which are typical for many geological settings and problems. Unless otherwise  
 199 explicitly stated they are kept constant throughout this study, but frequently we will vary  
 200 one or more parameter systematically while maintaining the others as listed in Table 1.

## 201 3. Linear stability analysis

202 In this section we investigate the linear stability of steady state sliding at slip velocity  
 203  $V_0$  to small Fourier mode perturbations. We assume that inertial effects can be neglected,  
 204 which is valid if  $GV_0/(2c_s a \sigma'_0) \ll 1$  where  $c_s$  is the S wave speed [36] (see also Table 1). The  
 205 linear poroelastic solution developed in Section 2.2 is utilized to describe perturbations about  
 206 the prestressed, steady sliding solution. We show that steady sliding with a low permeability  
 207 fault core is conditionally unstable for mildly rate-strengthening friction, in the sense that  
 208 that small amplitude perturbations can spontaneously grow to nucleate slip instabilities.  
 209 Later we demonstrate how these instabilities evolve in quasi-dynamic simulations, under  
 210 nonlinear friction effects, into propagating slow slip pulses.

### 211 3.1. Characteristic equation

212 Applying a Fourier transform in  $x$  and a Laplace transform in time, Eq. 19 is

$$\left(s + \frac{V_0}{L}\right) \hat{\tau} = \left[f_0 \left(s + \frac{V_0}{L}\right) - \alpha_{LD} s\right] \hat{\sigma}' + \left[\frac{a \sigma'_0}{V_0} s^2 + \frac{(a - b) \sigma'_0}{L} s\right] \hat{\delta}. \quad (23)$$

213 Next we insert the linear poroelastic relations, Eqs. 14 and 15, into Eq. 23 and obtain  
 214 the characteristic equation. Assuming slip localization on the  $y > 0$  side of the interface  
 215 (Fig. 1b), such that  $\hat{\sigma}' = -\hat{p}^+$ , the characteristic equation is

Table 1: Reference parameters

Symbol	Description	Value
<i>Material properties</i>		
$B$	Skempton's coefficient	0.6
$\nu$	Drained Poisson's ratio	0.25
$\nu_u$	Undrained Poisson's ratio	0.35
$G$	Shear modulus	30 GPa
<i>Friction</i>		
$L$	Characteristic state evolution distance	10 $\mu\text{m}$
$a$	Rate dependence of friction	0.01
$a - b$	Degree of rate-strengthening	$1.5 \times 10^{-4}$
$\alpha_{LD}$	Linker-Dieterich constant [43]	0
$V_0$	Steady state sliding velocity	$10^{-9}$ m/s
$f_0$	Steady state coefficient of friction at $V_0$	0.6
$\sigma'_0$	Initial effective normal stress	50 MPa
<i>Nondimensional parameters</i>		
$\gamma$	Lateral diffusion stabilization	0.08
$\epsilon$	Across fault flow stabilization (Eq. 28)	0
<i>Physical scales — dependent on parameters above</i>		
$\lambda_c$	Approximate preferred wavelength (Eq. 25)	
$v_p$	Phase velocity of $\lambda_c$ (Eq. 26)	
$r$	Growth rate of $\lambda_c$ (Eq. 27)	

$$\frac{a\sigma'_0}{V_0}s^2 + \left[ \frac{(a-b)\sigma'_0}{L} + \frac{G|k|H_1(s,k)}{2(1-\nu_u)} - ik\frac{BG}{3}\frac{1+\nu_u}{1-\nu_u}(f_0 - \alpha_{LD})H_2(s,k) \right] s + \frac{V_0}{L} \left[ \frac{G|k|H_1(s,k)}{2(1-\nu_u)} - ik\frac{BG}{3}\frac{1+\nu_u}{1-\nu_u}f_0H_2(s,k) \right] = 0. \quad (24)$$

216 The undrained limit, which was previously described ( $ck^2/s \rightarrow 0$  and  $\mathcal{F} < \infty$ ), is attained  
 217 from Eq. 24 by setting  $H_1 = H_2 = 1$ .

### 218 3.2. Undrained limit

219 In order to gain insight into the stability of the fault we solve Eq. 24 for  $s(k)$  with  
 220 near-rate-neutral friction,  $a - b = O(10^{-4})$  (Fig. 2). The figure reveals that a range of  
 221 wavelengths is linearly unstable to small perturbations at mildly rate-strengthening friction.  
 222 Unlike instabilities at the interface of two identical elastic half-spaces, there is a wavelength  
 223 of maximum growth rate, which we will refer to as the **preferred wavelength**, noting  
 224 that both larger and smaller wavelengths are stable. The stability at large wavelengths  
 225 suggests pulse-like propagation, rather than crack-like expansion of slip instabilities. Further  
 226 suggesting pulse-like behavior is the directional dependence of the solutions to Eq. 24,  
 227 which assumes slip localization on  $y > 0$  side of the fault core. The equation predicts that  
 228 a perturbation with  $k > 0$  (propagating to the right, as shown in Fig. 1c) experiences  
 229 pore pressure changes that can overcome the otherwise stabilizing effects of mildly rate-  
 230 strengthening friction, but perturbations with  $k < 0$  (i.e., propagating to the left, not  
 231 shown) have pressure changes that further stabilize sliding, due to the sign change in pore  
 232 pressure (Eq. 15). This gives rise to directionality and determines the pulse propagation  
 233 direction. For localization on the  $y < 0$  side of the fault core, the characteristic equation  
 234 is attained by changing the sign of the pore pressure terms (that is changing  $-ik \rightarrow ik$  in  
 235 Eq. 24). Then perturbations with  $k < 0$  can be unstable under rate-strengthening friction,  
 236 but  $k > 0$  perturbations are always stable. This suggests that pulses may propagate in both  
 237 directions on the same fault depending on where localization occurs. However, one direction  
 238 may be favored if there are additional elastic bimaterial effects, as we discuss later.

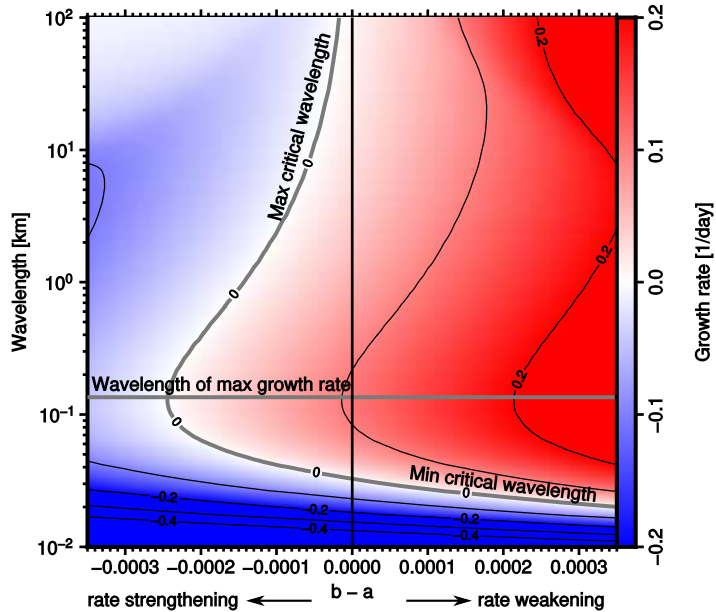


Figure 2: Growth/decay rate from linear stability analysis in the undrained limit, with the gray contour marking neutral stability. For rate-weakening friction ( $b - a > 0$ ), all wavelengths greater than a critical wavelength are unstable. For rate-strengthening friction ( $b - a < 0$ ), instability occurs for a range of wavelengths provided that  $a - b$  is sufficiently small. The wavelength of maximum growth rate, also marked, is relatively independent of  $a - b$ . Parameters given in Table 1. See also Fig. 3.

239 If slip localizes in the fault core, but away from its boundaries, then the magnitude of  
 240 the pore pressure perturbation is reduced (Fig. 1b). Exactly in the center of the fault core  
 241 there is no perturbation in pore pressure, and this configuration is therefore linearly stable  
 242 at mildly rate-strengthening friction. The corresponding solution can be attained from Eq.  
 243 24 by setting  $H_2 = 0$ .

244 For simplicity and insight, we provide approximate expressions valid in the undrained  
 245 limit (negligible fluid flow) near rate-neutral friction ( $a \sim b$ ) and assuming  $\alpha_{LD} = 0$ . We  
 246 observed from numerical solutions to Eq. 24 that  $|\text{Re}(s)| \ll |\text{Im}(s)|$  near rate-neutral fric-  
 247 tion, meaning that the growth rate of perturbations is much smaller than their angular  
 248 frequency. Given these observations we assume the second order  $\text{Re}(s)^2$  term may be ig-  
 249 nored which results in explicit closed form solutions for  $\text{Re}(s)(k)$  and  $\text{Im}(s)(k)$ , valid if

250  $\text{Re}(s)(k) \ll \text{Im}(s)(k)$ . Then solving  $d\text{Re}(s)(k)/dk = 0$  for  $k$  gives an approximate expres-  
 251 sion for the preferred wavenumber. Substituting this approximate expression in  $\text{Re}(s)(k)$  and  
 252  $\text{Im}(s)(k)$  provides growth rate and angular frequency evaluated at the preferred wavenum-  
 253 ber. However, in spite of the  $|\text{Re}(s)(k)| \ll |\text{Im}(s)(k)|$  assumption, these expressions are too  
 254 complicated to provide insight into first order effects. We perform a Taylor expansion to  
 255 leading order in  $Bf_0$ , recognizing that under most conditions  $Bf_0$  is smaller than unity. This  
 256 approximation of the preferred wavelength is

$$\lambda_c \equiv \frac{9\pi LG}{\sigma'_0 a(1 + \nu_u)^2(1 - \nu_u)(Bf_0)^2}. \quad (25)$$

257 Slip pulses propagate along the interface in the direction of strength reduction (Fig. 1c),  
 258 and we gain insight into their propagation speed by deriving the approximate phase velocity,  
 259  $v_p = -\text{Im}(s)/k$ , at  $\lambda_c$ :

$$v_p \equiv \frac{3V_0 G}{2\sigma'_0 a(1 + \nu_u)(1 - \nu_u)Bf_0}. \quad (26)$$

260 The approximate growth rate of  $\lambda_c$  is

$$r \equiv \frac{V_0}{L} \left[ \frac{(Bf_0)^2}{18} (\nu_u + 1)^2 + \frac{b - a}{2a} \right]. \quad (27)$$

261 Equation 27 also quantifies the maximum rate-strengthening  $a - b$  that can be destabilized  
 262 by effective normal stress changes:  $(a - b)_{\text{crit}} \approx a(1 + \nu_u)^2(Bf_0)^2/9$ .

### 263 3.3. Correspondence between undrained poroelastic and elastic bimaterial problems

264 The linear stability results for the elastic bimaterial problem[36, 38] are mathematically  
 265 identical to those describing the undrained poroelastic problem (where  $H_1 = H_2 = 1$ ), with  
 266 the substitution  $G/(1 - \nu_u) \rightarrow M$  and  $2B(1 + \nu_u)/3 \rightarrow \beta$ , where  $M$  and  $\beta$  are elastic  
 267 bimaterial parameters defined by Rice et al.[36] to quantify elastic moduli and material con-  
 268 trast, respectively. Apparently, though, the connection between slow slip pulses and stability  
 269 characteristics was overlooked in previous studies. The correspondence between undrained  
 270 poroelastic sliding and the elastic bimaterial sliding goes beyond the stability characteris-  
 271 tics since the relationships between sliding and fault stresses are also identical through the  
 272 aforementioned substitution. For example, Eq. 14 becomes  $\hat{\tau} = -(M/2)|k|\hat{\delta}$  and Eq. 15



273 gives  $\hat{\sigma}' = (M\beta/2)ik\hat{\delta}$ , which demonstrates a direct parallel between the two problems, at  
 274 least when inertial and elastodynamic effects are negligible. This correspondence between  
 275 quasi-static sliding of elastic bimaterial and undrained poroelastic interfaces has not been  
 276 pointed out before to the best of our knowledge.

277 Note that for natural faults,  $\beta$  is typically less than 0.1 [36], while  $B \approx 0.5$  to 0.9 [41].  
 278 Thus we conclude that destabilization by poroelastic effects is more likely to cause slow slip  
 279 instabilities than elastic bimaterial effects on rate-strengthening faults, justifying our focus  
 280 on the poroelastic instability numerical simulations.

281 It is worth noting that mildly rate-strengthening friction can also be destabilized during  
 282 sliding on a interface between two identical elastic materials when the system lacks geometric  
 283 reflection symmetry, a situation that can arise, for example, when sliding two plates of  
 284 different thickness across one another[40]. However, these systems will possess additional  
 285 length scales associated with the geometry (e.g., plate thickness) and thus are not generally  
 286 mathematically equivalent to undrained poroelastic or elastic bimaterial sliding.

### 287 *3.4. Stabilizing effects of diffusion and fluid flow*

288 Both lateral diffusion and diffusion across the fault core will act to equilibrate poroelastic  
 289 pressure changes. If this equilibration process occurs sufficiently fast, as compared to the  
 290 growth time of the instability described in previous sections, then sliding will be stabilized.  
 291 Here we identify two nondimensional parameters,  $\gamma$  and  $\epsilon$ , that quantify the importance of  
 292 lateral and across-fault diffusion, respectively.

293 To determine the time scales over which pressure equilibration occurs, we examine Eq.  
 294 17 that expresses pore pressure change on the fault. Pressure change vanishes if the function  
 295  $H_2(s, k) \rightarrow 0$ . This can occur if either  $ck^2/s$  (comparing Laplace parameter  $s$  to the diffu-  
 296 sion time along the fault,  $(k^2c)^{-1}$ ) or  $\mathcal{F}$  (quantifying across-fault pressure equilibration) is  
 297 sufficiently large. Eq. 17 also reveals that the relative magnitude of  $ck^2/s$  and  $\mathcal{F}$  determines  
 298 which equilibration mechanism is dominant. If  $\mathcal{F}/\sqrt{1 + s/ck^2} \ll 1$ , then the fault core  
 299 can be regarded as impermeable, so pressure equilibration occurs by lateral flow parallel to  
 300 the fault. This will stabilize the system if  $ck^2/s$  is sufficiently large. We thus deduce that

301 instability requires that both  $\mathcal{F}/\sqrt{1+s/ck^2} \ll 1$  and  $ck^2/s \ll 1$ . However,  $s = s(k)$  is  
 302 generally complex and therefore inappropriate for use in defining dimensionless parameters.

303 We therefore nondimensionalize  $s$  and  $k$ , first by generic time and length scales, then  
 304 later by selecting these scales as those characterizing the maximum growth rate instability  
 305 under undrained conditions. Let  $s^*$  and  $k^*$  be characteristic growth rate and wavenumber,  
 306 respectively, such that in a relevant range the nondimensional growth rate  $\tilde{s} = s/s^*$  and  
 307 wavenumber  $\tilde{k} = k/k^*$  are both of order unity. It follows that  $ck^2/s = c(k^*)^2/s^* \times \tilde{k}^2/\tilde{s}$ ,  
 308 where  $\tilde{k}^2/\tilde{s}$  is of order unity. The nondimensional parameter is identified as  $\gamma \equiv c(k^*)^2/s^*$ ,  
 309 which is the ratio the time scale of lateral diffusion,  $[(k^*)^2c]^{-1}$  and the time scale of the  
 310 instability  $(s^*)^{-1}$ . Instability is promoted by small values of  $\gamma$ .

311 Across-fault diffusion is negligible relative to lateral diffusion when  $\mathcal{F}/\sqrt{1+s/ck^2} \ll 1$ .  
 312 To quantify the relative importance of these processes, we write  $\mathcal{F} = 2\kappa_c/(\tilde{k}\kappa wk^*)$ , from  
 313 which we identify  $\psi \equiv 2\kappa_c/\kappa wk^*$ . Now  $\sqrt{1+s/ck^2} \sim \sqrt{1+1/\gamma}$  and if  $\gamma \ll 1$ , as required  
 314 for instability, then  $\sqrt{1+1/\gamma} \approx \sqrt{1/\gamma}$ . We then identify the nondimensional ratio that  
 315 characterizes the competition between across-fault and lateral diffusion:

$$\epsilon \equiv \psi\sqrt{\gamma} = \frac{2\kappa_c}{\sqrt{\kappa w}} \sqrt{\frac{M}{s^*}}. \quad (28)$$

316 Instability is promoted by small values of  $\epsilon$ .

317 Next we take  $s^* = r$  in Eq. 27, with  $a = b$  (rate-neutral friction), and  $k^* = 2\pi/\lambda_c$  from  
 318 Eq. 25. These scales are used to nondimensionalize results in Fig. 3. Fig. 3a and b show  
 319 how stability at rate-neutral friction changes by systematically varying  $\gamma$  and  $\epsilon$ , respectively,  
 320 and solving Eq. 24 for  $s = s(k)$ . This comparison reveals that the choice of characteristic  
 321 scales and nondimensional parameters is appropriate and the conditions  $\epsilon, \gamma \ll 1$  yield the  
 322 undrained response.

323 If  $\epsilon \ll 1$ , then the dominant diffusion mechanism is lateral and the fault core is effectively  
 324 impermeable on relevant time scales, a necessary but not sufficient condition for instability.  
 325 It is furthermore necessary that  $\gamma$  be sufficiently small such that lateral diffusion cannot  
 326 stabilize the nucleation process. Interestingly, the condition on  $\gamma$  is far less restrictive than  
 327  $\gamma \ll 1$ , as Fig. 3a shows instability even for  $\gamma$  several orders of magnitude larger than unity.

328 In addition, Fig. 3c compares the approximate solutions of Eqs. 25, 26, and 27, developed  
329 under the assumption that  $Bf_0 \ll 1$ , with numerical solutions to Eq. 24, showing good  
330 agreement even though  $Bf_0 = 0.36$ . There is a clear peak in growth rate ( $\text{Re}(\tilde{s})$ ) in the  
331 vicinity of the preferred wavelength for both rate-strengthening and rate-weakening friction.  
332 However, at increasingly rate-weakening friction a more rapidly growing instability occurs  
333 at larger wavelengths. It is worth noting that in nature we could expect  $Bf_0$  as large as  
334  $0.5 - 0.7$ , where the premise of the expansion in Eqs. 25, 26, and 27 is questionable. In that  
335 case the reader may want to simply solve Eq. 24 to get a more accurate answer. However,  
336 we have found that the first order expansions are roughly correct up to a factor of 2 for  
337  $Bf_0 \approx 0.6$  and may thus still be useful.

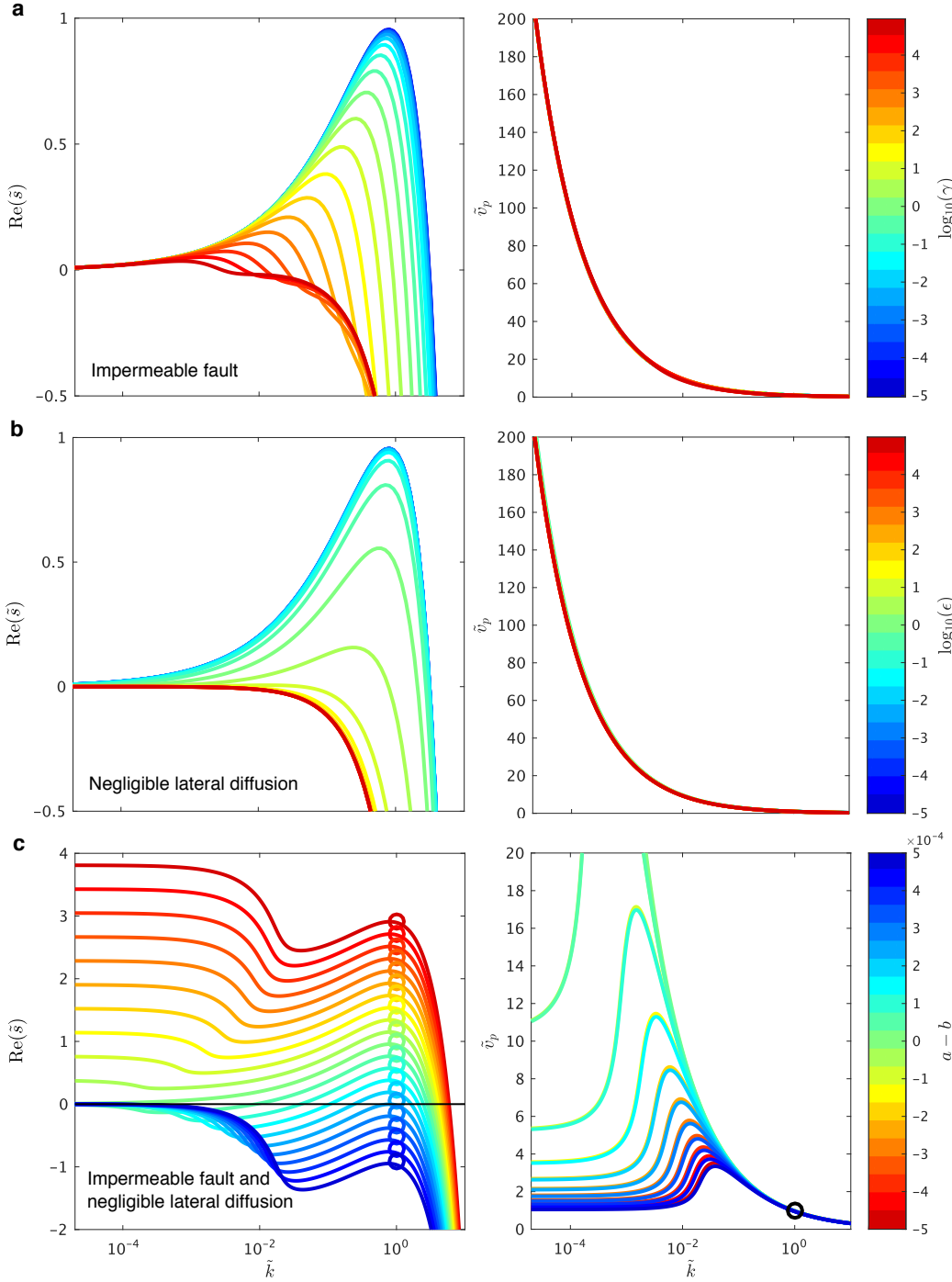


Figure 3: Linear stability analysis results showing nondimensional growth rate  $\text{Re}(\tilde{s})$  and phase velocity  $\tilde{v}_p$  as a function of nondimensional wavenumber  $\tilde{k}$ . We nondimensionalize  $k$ ,  $v_p$ , and  $\text{Re}(s)$  by Eqs. 25, 26, and 27, respectively, with  $a = b$ . **a**, Fault core is impermeable ( $\psi = 0$ , and thus  $\epsilon = 0$ ) but lateral diffusion is allowed by changing  $\gamma$ . Friction is rate-neutral ( $a = b$ ). Phase velocity is largely independent of  $\gamma$ . **b**, Negligible lateral diffusion ( $\gamma = 2.4 \times 10^{-6}$ ) but  $\epsilon$  is varied to explore effects of across-fault flow. Friction is rate-neutral. **c**, Effectively undrained limit ( $\gamma = 2.4 \times 10^{-6}$ ,  $\epsilon = 0$ ) for various  $a - b$ . Circles indicate approximate values given by Eqs. 25, 26, and 27. Phase velocity depends on  $a - b$ , although near the preferred wavelength it is relatively constant. Different values of  $\gamma$  and  $\epsilon$  are explored by altering the mobility parameters  $\kappa$  and  $\kappa_c$ , respectively; other parameters are as listed in Table 1.

## 338 4. Numerical simulations

339 In the previous section we presented a linearized analysis that is only strictly valid for  
340 small perturbations around steady state. Now we explore numerically how instabilities  
341 evolve once nonlinear effects become important.

### 342 4.1. Problem statement and numerical methodology

343 The linear poroelastic equations are solved using a finite difference method with summation-  
344 by-parts properties [44, 45]. Boundary conditions are enforced weakly using carefully chosen  
345 penalty terms [46] such that numerical stability can be established using the energy method.  
346 The numerical strategy follows [47] closely, using their fluid content formulation, but has  
347 been extended to allow for stretched grids and enforce displacement boundary conditions  
348 without approximating them by Robin conditions.

349 Simulations are conducted with a uniform grid spacing and periodic boundary conditions  
350 in the  $x$  direction. The sliding interface ( $y = 0$ ) is assumed to be impermeable ( $\partial p/\partial y = 0$ ).  
351 Assuming antisymmetry of displacement component  $u_x$  and symmetry of  $u_y$  about  $y = 0$ ,  
352 together with no opening or interpenetration of material across the interface, there is no  
353 change in total normal stress  $\sigma_{yy}$  on the interface. We exploit these symmetries to model only  
354 the top poroelastic block, replacing interface conditions with boundary conditions. The third  
355 condition on the interface sets shear traction equal to the rate-and-state frictional strength.  
356 The top boundary, parallel to the sliding interface, is placed at  $y = 7\lambda_c$ . Boundary conditions  
357 on it are displacement at constant rate ( $u_x = V_0 t/2$ ), no normal displacement ( $u_y = 0$ ), and  
358 no fluid flow through the boundary ( $\partial p/\partial y = 0$ ). Results are relatively independent of the  
359 latter two conditions if the domain is sufficiently large. For computational efficiency, we  
360 applied a coordinate transformation in the  $y$  direction such that the grid spacing is finer  
361 near the fault and becomes ten times larger over a distance of  $1.5\lambda_c$ . The domain size in  
362 the  $x$  direction ranges from 17 to 50 times  $\lambda_c$ . Unless stated otherwise, we keep  $\gamma = 0.08$  by  
363 varying the hydraulic diffusivity  $c$ , which ranges from  $5 \times 10^{-5}$  to  $0.02 \text{ m}^2/\text{s}$ . In comparison,  
364 relevant types of rock can have  $c$  in the range of  $10^{-5}$  to  $1 \text{ m}^2/\text{s}$  [41]. In the simulations we  
365 only explore the limit of a fully impermeable fault ( $\epsilon = 0$ ). With regard to initial conditions,

366 the block is subjected to shear strain such that the shear stress at the frictional interface is  
367  $\tau = f_0\sigma'_0$ , corresponding to steady sliding at slip rate  $V_0$  if the initial state variable is also  
368 set equal to its steady state value. The far-field constant velocity loading ensures that as the  
369 fault slides at steady state the block moves without additional straining, thus maintaining  
370 constant shear stress at the interface. Perturbations to initial state variable are added to  
371 trigger departures from steady sliding and potentially sliding instability.

#### 372 *4.2. Spontaneous formation of slip pulses*

373 Simulations with fully nonlinear rate-and-state friction response at mildly rate-strengthening  
374 friction support the interpretations of the linear stability analysis. Linear stability at larger  
375 wavelengths leads to slip pulses (Figs. 1c, 4, 5). Furthermore, we find that from slight  
376 white noise perturbations to a fault driven at steady state, there is selection of a wavelength  
377 of maximum growth rate which propagates along the fault with phase velocity; both the  
378 wavelength and phase velocity are in agreement with linear theory (Fig. 4).

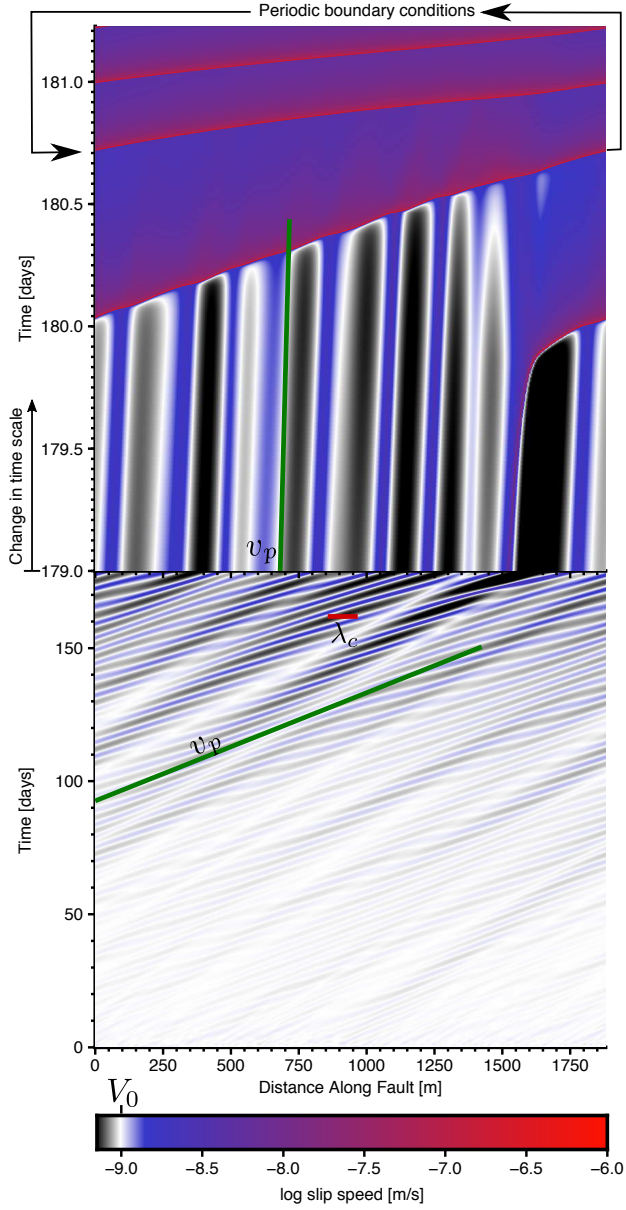


Figure 4: Evolution of slip rate in a simulation of sliding between proelastic blocks with rate-strengthening friction. Certain wavelength perturbations, seeded from random initial state conditions, grow in accordance with the linearized analysis until nonlinearities trigger slip pulse formation around 179-180 d (note change in time axis at 179 d). Continued propagation of the slip pulse smooths heterogeneities, and the system enters a steady, inhomogeneous sliding state with one active slip pulse (see also Fig. 1c). The approximate preferred wavelength  $\lambda_c$  (Eq. 25) and associated phase velocity  $v_p$  (Eq.26) from the linearized analysis are shown with red and green lines, respectively. Parameters given in Table 1.

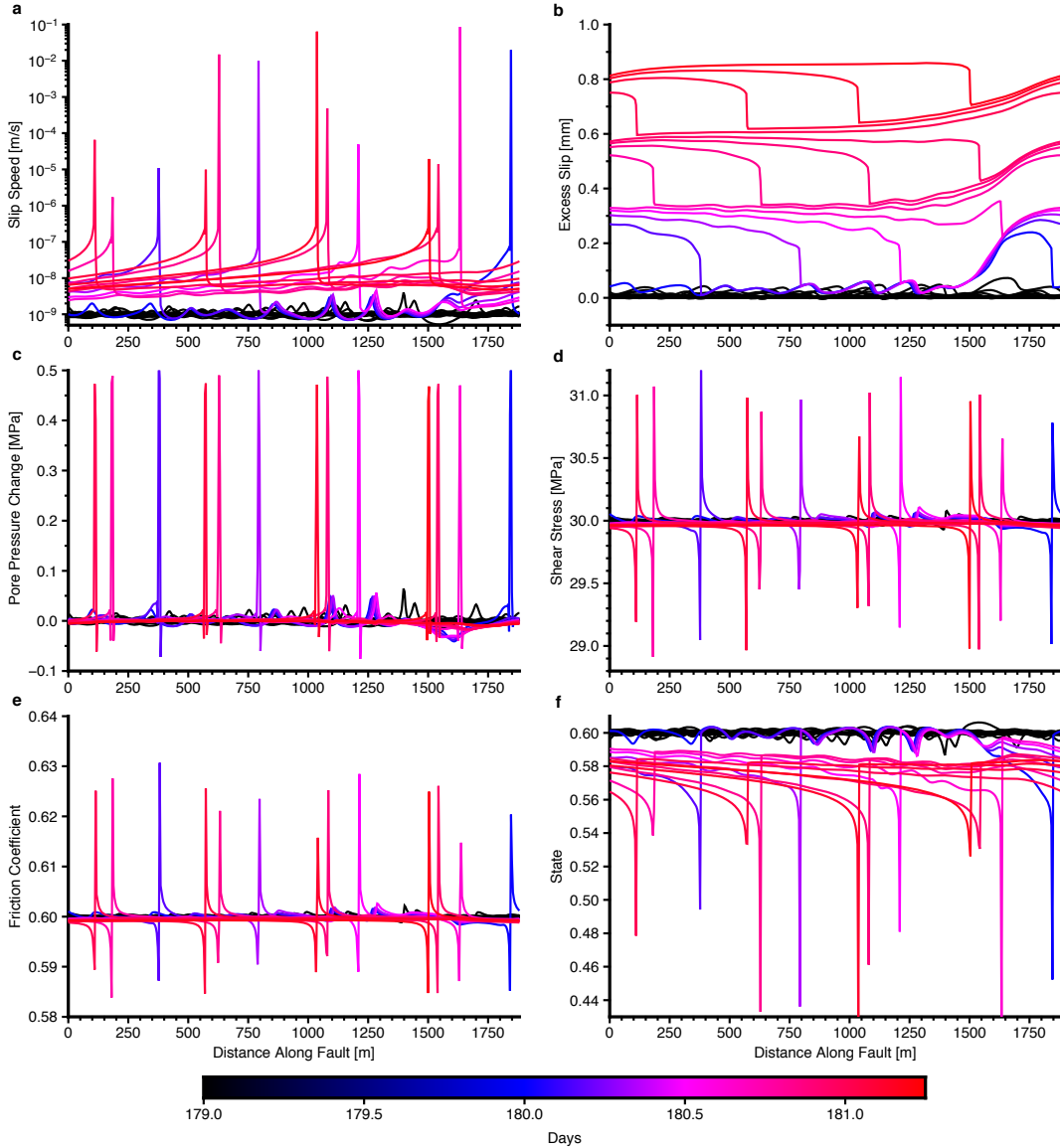


Figure 5: Snapshots of various fields on the sliding interface from the numerical simulation shown in Fig. 4. Lines that correspond to earlier times than day 179 (roughly the onset of the slip pulse) are black.

379 From investigating snapshots of various fields on the slip surface in the simulation in Fig.  
 380 4 we observe several important characteristics of the slip pulses. Firstly, the pulse slip in  
 381 excess of steady state sliding is only about 0.3 mm (Fig. 5b). Secondly, the shear stress drop  
 382 is only a fraction of a mega pascal. These are commonly observed characteristics of slow  
 383 slip[48], which distinguish slow slip in nature from earthquakes. However these potential



384 observables depend on the assumed parameters, this dependence is explored in Fig. 6.

### 385 *4.3. Slip pulse characteristics*

386 Results from analysis of the linearized problem help explain parameter controls on pulse  
387 length  $\Lambda_c$  and propagation speed  $V_p$  in the fully nonlinear simulations, though we find that  
388 for the chosen parameters, slip pulses are 10 to 100 times longer and faster than predicted by  
389 the linear theory. We quantify the characteristics of slip pulses in our numerical simulations  
390 (Fig. 6) as follows. The simulation domain is  $50\lambda_c$  in  $x$  and  $7\lambda_c$  in  $y$ . A region along the  
391 fault of length  $3.5\lambda_c$  is perturbed about steady sliding to trigger instability. A slip pulse  
392 forms and propagates into the unperturbed region, and its length  $\Lambda_c$  and propagation speed  
393  $V_p$  are measured. An example of this type of simulation is shown in Fig. 7. We define  $\Lambda_c$  as  
394 the distance from the peak slip rate to where the slip rate has decayed to  $1.5V_0$ . The excess  
395 slip is measured as the maximum average slip over the whole simulation domain in excess  
396 of the background sliding. Similarly, the stress drop is measured as the maximum spatially  
397 averaged shear stress drop during the simulation relative to the steady state stress, that is,

$$\Delta\tau = \max_t \frac{1}{l} \int_0^l [f_0\sigma'_0 - \tau(x, t)] dx, \quad (29)$$

398 where  $l$  is the length of the simulation domain. These definitions of stress drop and excess  
399 slip, while different than standard seismological definitions, are appropriate for a spatially  
400 periodic system with steady external loading. Some variability in slip pulse characteristics  
401 is observed when changing how the fault is initially perturbed or altering the domain size  
402 (which can affect how many pulses nucleate and are simultaneously active). However, this  
403 variability is relatively minor, and the approach outlined here gives consistent results that  
404 can be compared in a meaningful manner.

405 Generally speaking, we find that the expressions for preferred wavelength and associated  
406 phase velocity from the linearized analysis correctly predict parameter combinations that  
407 determine slip pulse length and propagation speed. Furthermore, we see that stress drop  
408 and excess slip depend on assumed parameters (Fig. 6c and d). For example, larger  $L$   
409 results in larger excess slip, and higher  $\sigma'_0$  results in a higher stress drop.

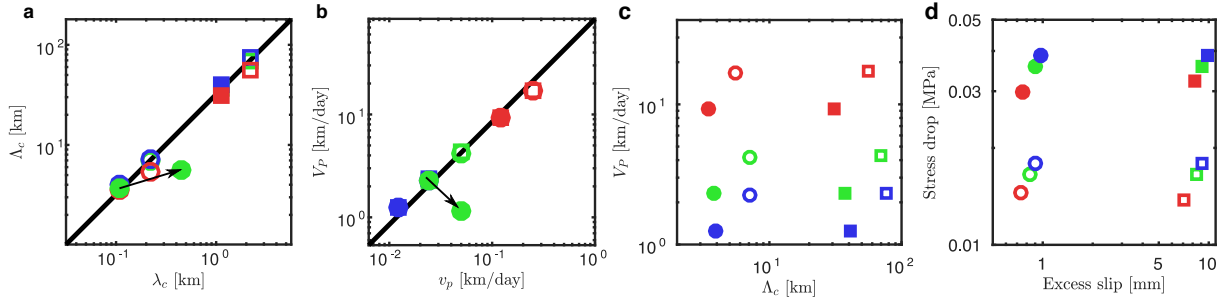


Figure 6: Comparison of simulated slip pulse characteristics with linear theory. **a** Slip pulse length  $\Lambda_c$  is proportional to the preferred wavelength  $\lambda_c$  from the linear theory (Eq. 25), with approximate relationship  $\Lambda_c \approx 32\lambda_c$  (black line). **b** Slip pulse propagation speed  $V_p$  is proportional to phase velocity of the preferred wavelength from the linear theory (Eq. 26), with approximate relationship  $V_p \approx 85v_p$  (black line). **c** Cross-plot of slip pulse length and propagation speed. **d** Same as **c** for stress drop and slip. In all panels, symbols/colors indicate parameter variations: Circles,  $L = 10 \mu\text{m}$ ; squares  $L = 100 \mu\text{m}$ . Filled symbols,  $\sigma'_0 = 50 \text{ MPa}$ ; open symbols,  $\sigma'_0 = 25 \text{ MPa}$ . Blue,  $V_0 = 5 \times 10^{-7} \text{ mm/s}$ ; green,  $V_0 = 10^{-6} \text{ mm/s}$ ; red,  $V_0 = 5 \times 10^{-6} \text{ mm/s}$ . Arrow shows how changing  $Bf_0 = 0.36$  to  $0.18$  for one simulation alters the scaling between linear and nonlinear characteristics, suggesting more complex dependence on  $Bf_0$  than predicted by linear theory. Other parameters given in Table 1.

#### 4.4. Mildly rate-weakening friction

In this study we have focused primarily on simulations with mildly rate-strengthening friction, but it is worth commenting on the character of nonlinear slip behavior for rate-weakening friction. In Figs. 2 and 3c we observe at mildly rate-weakening friction ( $b - a = O(10^{-4})$ ) that very large wavelengths become unstable. These unstable wavelengths are relatively independent of poroelastic processes and will remain unstable even if  $\gamma$  or  $\epsilon$  are large. These wavelengths will, therefore, likely cause seismic or inertially limited events, assuming that the fault is sufficiently large to host such wavelengths. However, it is not clear at mildly rate-weakening friction if all wavelengths will generate seismic events or manifest as slow slip pulses. Preliminary nonlinear simulations under mildly rate-weakening friction (not shown here) suggest that mildly rate-weakening friction also produces stable slow slip pulses. These simulations, done with periodic boundary conditions, may simply not have a large enough domain to activate wavelengths capable of producing seismic events. We suggest that a fruitful topic of future research may focus on investigating the partition

424 of wavelengths capable of producing seismic events or slow slip. This could shed light on  
425 the potential transition from slow slip to seismic events.

## 426 **5. Discussion**

### 427 *5.1. Experiments on hydrogels*

428 Laboratory hydrogel experiments[8, 7, 9, 10, 11] have demonstrated sliding instabilities  
429 and slow slip pulses that are possibly explained by our mechanism. A poroelastic gel block  
430 is slid across a glass substrate, which may activate both poroelastic and elastic bimaterial  
431 destabilizing effects. Consistent with our predictions, steady sliding transitions sponta-  
432 neously into slip pulses that advance, in the direction of motion of the gel block, at speeds  
433 much slower than wave speeds but several orders of magnitude faster than the driving speed  
434  $V_0$ . In particular, experiments in an annular geometry[7], somewhat like our simulations  
435 with periodic boundary conditions, show evolution to a steady, inhomogeneous sliding state  
436 (Fig. 7) with direct proportionality between pulse speed and  $V_0$  (c.f., Eq. 26). Moreover,  
437 the pulse length is independent of  $V_0$  (c.f., Eq. 25).

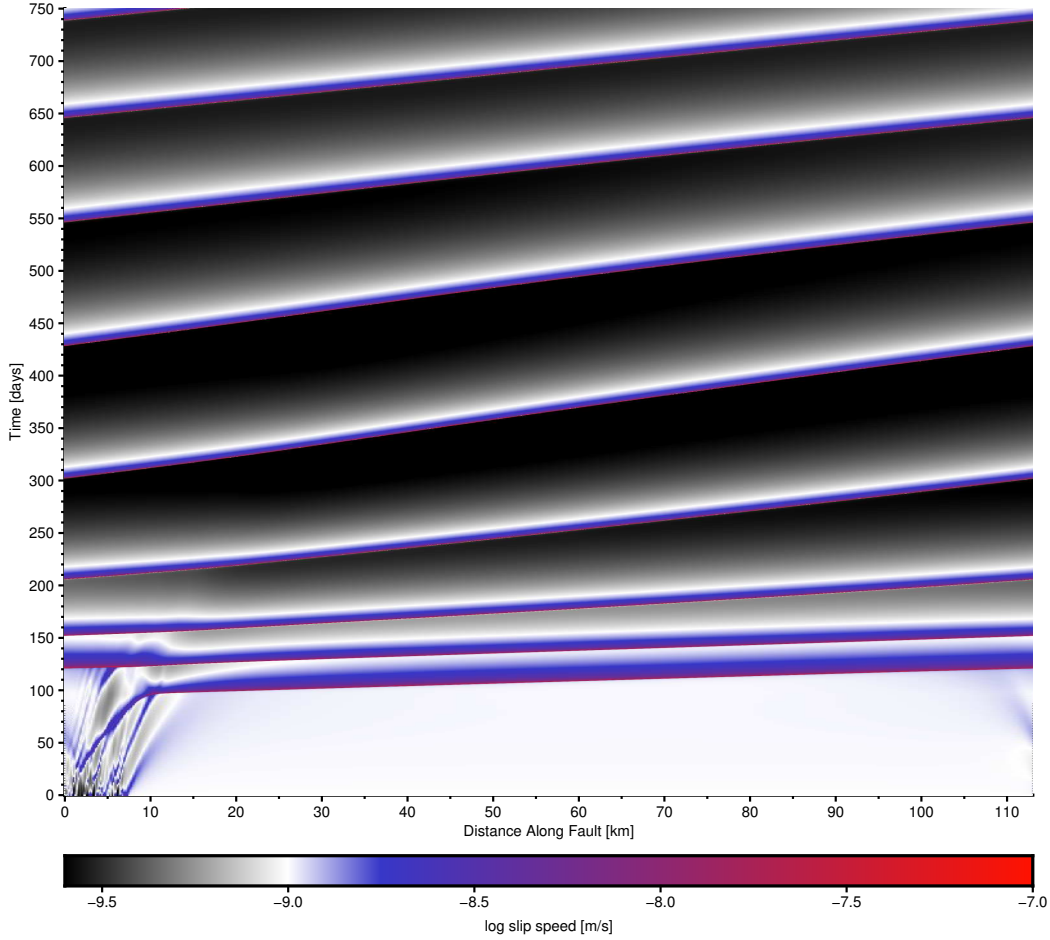


Figure 7: Evolution of slip rate in a simulation, similar to that shown in Figs. 4 and 5, but started from spatially localized perturbations in state ( $0 < x < 3.5\lambda_c$ ). The simulation is run for longer, until the system evolves to steady, inhomogeneous sliding with one (or more) active slip pulses. This is an example of a simulation that is used to characterize slip pulse characteristics reported in Fig. 6. Parameters here correspond to open green squares in Fig. 6.

438 Sliding of gel on glass features a dramatic contrast in elastic moduli, and thus one  
 439 might expect that the elastic bimaterial effect is dominant. However, many gels are nearly  
 440 incompressible. Experiments on gelatine suggest that the undrained Poisson's ratio  $\nu_u \approx 0.5$ ,  
 441 but a drained value  $\nu \approx 0.45$  [49]. This indicates that the elastic bimaterial coefficient  $\beta$   
 442 (defined in section 3.3 and quantifying normal stress changes from nonuniform slip) ranges  
 443 from 0 to 0.1 [36], with  $\beta = 0$  if the gel is completely incompressible. In contrast, Skempton's

444 coefficient  $B \approx 1$  (Eq. 5) and hence the poroelastic coupling may cause larger changes in  
445 effective normal stress than elastic bimaterial effects if our treatment of the interface effective  
446 normal stress is valid for these systems. It may be that both effects contribute, as well as  
447 coupling of slip to normal stress that arises from the lack of geometric reflection symmetry  
448 [40] that is likely present in many hydrogel experiments.

449 One important observation of gel-on-glass sliding is that above a critical loading velocity  
450 a homogeneous sliding mode set in and no slip pulses nucleate [9, 10]. An explanation of this  
451 critical velocity has been offered using the framework of fracture mechanics, a linear viscous  
452 friction law, and by drawing a comparison to first-order phase transitions[50]. Our analysis  
453 does not directly explain this critical loading velocity, but it is worth noting that hydrogels  
454 have a complex and often non-monotonic frictional strength at different slip speeds[51]. This  
455 suggests that  $a - b$  depends on velocity, and if the gel becomes sufficiently rate-strengthening  
456 at a certain velocity, the instability we have identified would be suppressed.

457 We recognize that hydrogel experiments demonstrate several qualitative characteristics  
458 that agree with our theoretical findings, but that the mechanical response of hydrogels  
459 involves processes not present in our model. These include viscoelasticity, spatial finiteness,  
460 edge effects, and possibly interface adhesion. Further study is needed to determine if we have  
461 identified the right destabilization mechanism that manifests in these hydrogel experiments.

## 462 *5.2. Potential application to subduction zone slow slip*

463 We propose that our mechanism could be applied to subduction zone slow slip. However,  
464 any true 3-D manifestations of this mechanism are hypothetical at this point and require  
465 further study. Direct application of our 2-D simulations to subduction zones is complicated  
466 by the 3-D nature of subduction slow slip, where slower migration along strike (i.e., in  
467 the invariant dimension in our simulations) is interspersed with faster along-dip transients  
468 [52, 53]. Our instability mechanism would act only in the along-dip (mode II) direction,  
469 but along-strike (mode III) migration might be driven by stress transfer from currently  
470 slipping sections of the interface that activates instability in adjacent sections (Fig. 8).  
471 Furthermore, instabilities might also arise behind the main slip front and are predicted to

472 propagate along-dip at higher velocities due to the accelerated sliding rate (c.f., Eq. 26).  
473 Similar ideas have been previously proposed based on tremor observations and geodetic  
474 modeling that suggest the large scale slow slip is, in reality, the manifestation of many slow  
475 transients[54]. It is worth noting that tremors migrate both up and down dip in subduction  
476 zones[53]. The poroelastic mechanism can explain both directions since the directionality  
477 is simply determined by in which side of the fault core slip localization occurs. However, a  
478 bimaterial destabilizing mechanism cannot explain migration in both directions.

479 The migration of tremor along the dip direction in subduction zones has been inferred to  
480 be faster than in simulations in the paper (Fig. 6). For example, in Japan they are around  
481 25 to 250 km/h compared to the along-strike migration of  $\sim 10$  km/d [55]. Similar migration  
482 speeds of low frequency tremor are also observed on strike slip faults in the in-plane direction  
483 of sliding[56].

484 The mechanism we have presented can potentially explain the along-dip migration rates  
485 as large as observed in Japan and elsewhere if effective normal stress is sufficiently low.  
486 Indeed, previous authors have suggested that the effective normal stress may be around 0.1  
487 MPa[57]. The effective normal stress in most simulations in this study has been  $\sim 50$  MPa,  
488 which results in propagation speed of about  $\sim 0.1$  km/h. This is  $\sim 100$  to  $1000$  times slower  
489 than the previously mentioned values for Japan. To test if comparable propagation speeds  
490 are observed in simulations at low effective normal stress, we ran two additional simulations  
491 at  $\sigma'_0 = 1$  MPa and  $0.1$  MPa, but otherwise with reference parameters in Table 1. The  
492 setup of these simulations is otherwise the same as in Section 4.3. The low effective stress  
493 simulations reveal  $V_P = 800$  km/d =  $33$  km/h for  $\sigma'_0 = 0.1$  MPa and  $V_P = 140$  km/d  
494 =  $5.8$  km/h for  $\sigma'_0 = 1$  MPa. This demonstrates that at low effective normal stress the  
495 simulations predict the right order of magnitude for the tremor migration speeds observed  
496 in nature. Furthermore, the simulations at low effective normal stress show that the inferred  
497 relationship  $V_p \approx 85v_p$  in Fig. 6 still holds reasonably well.

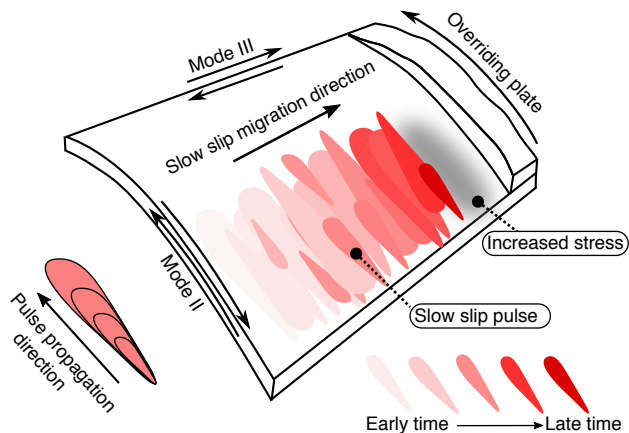


Figure 8: Schematic illustrating slip pulse instability in mode II direction, with mode III propagation driven by stress transfer.

498 Slow slip can also occur in the shallow parts of subduction zones, where experiment on  
 499 rock samples have confirmed rate-strengthening behavior [18]. These regions may combine  
 500 three processes that can destabilize rate-strengthening friction: poroelastic coupling, elastic  
 501 bimaterial contrast, and lack of geometric reflection symmetry[58, 40]. All these processes  
 502 may act simultaneously and should be considered in mechanical models of shallow subduction  
 503 zone processes such as slow slip events and tsunami earthquakes[59].

### 504 5.3. Further applicability

505 Our results might also apply to other problems. These include magnitude 7 slow slip  
 506 events of the Whillans Ice Plain[60], an Antarctic ice stream sliding on rate-neutral glacial  
 507 till[61], and slow, episodic advance of landslide masses[62]. Finally, in the context of reser-  
 508 voir geomechanics, if fault/fracture networks have heterogeneous frictional properties, the  
 509 advance of slow slip along a rate-strengthening fault might manifest as a swarm of small  
 510 earthquakes that migrates at relatively constant speed. Microseismic swarms accompanying  
 511 fluid injection (e.g., in oil/gas operations) can indeed migrate faster than can be explained  
 512 by pore pressure diffusion, possibly as a consequence of aseismic slip[6, 63]. Additionally,  
 513 the possibility of slow slip with rate-strengthening friction brings more consistency to the

514 hypothesized role of slow slip in reservoir stimulation (permeability enhancement by fluid  
515 injection and hydraulic fracturing)[5]. Reservoir rocks such as shales contain clays and  
516 organics that are experimentally linked to rate-strengthening behavior[17]. The reservoir  
517 setting might also be ideal for model validation through direct measurements of pore pres-  
518 sure changes on or adjacent to faults concurrently with slip using recently developed borehole  
519 instruments[6].

## 520 **6. Conclusions**

521 We have investigated spontaneously occurring sliding instabilities that occur with mildly  
522 rate-strengthening friction due to the coupling of slip and effective normal stress via two dif-  
523 ferent mechanisms. These instabilities are fundamentally different from standard earthquake-  
524 inducing instabilities with rate-weakening friction in that they are characterized by a pre-  
525 ferred wavelength having a maximum growth rate, with stability at both smaller and larger  
526 wavelengths, and a strong preference in propagation direction. Simulations with nonlinear  
527 rate-and-state friction show that these instabilities become slow slip pulses and are broadly  
528 consistent with many aspects of slow slip in nature, such as low stress drops and small  
529 slip distances. Furthermore, we find quantitatively similar propagation speeds of pulses  
530 as compared to tectonic tremor and geodetically inferred slow slip migration speeds under  
531 low effective normal stress conditions. We also observe qualitative consistency with slow  
532 slip pulses identified in experiments on hydrogels. We have proposed a conceptual model  
533 for how this type of instability might manifest in 3-D in a subduction zone setting, where  
534 unstable pulses propagate in the along-dip (mode II) direction and along-strike (mode III)  
535 migration is driven by secondary stress transfer due to cascading of pulses. However, we  
536 recognize that in order to make a full comparison to slow slip in experiments and geological  
537 settings we need to understand how the reported frictional instabilities manifest in three di-  
538 mensions. In summary, our work demonstrates how poroelastic and elastic bimaterial effects  
539 can destabilize mildly rate-strengthening sliding to generate slow slip events having features  
540 consistent with observations.



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550 [bitbucket.org/ericmdunham/poroelastic](https://bitbucket.org/ericmdunham/poroelastic).

## 551 Declaration of Interest

552 None

## 553 Appendix A. Poroelastic solution

554 This appendix outlines the derivation of the linear relations between shear stress change  
555 and slip and pore pressure change and slip that are implemented in deriving the characteristic  
556 equation 24.

557 We solve Eqs. 1 and 2 using the method of displacement functions[64, 65], which are a  
558 special case of the Biot potentials[66] applicable to plane strain problems. In order to find  
559 the displacement functions,  $\mathcal{S}$  and  $\mathcal{E}$ , we solve

$$\nabla^2 \mathcal{S} = 0, \tag{A.1}$$

$$\frac{\partial}{\partial t}(\nabla^2 \mathcal{E}) - c \nabla^4 \mathcal{E} = 0. \tag{A.2}$$

560 By Fourier transforming with respect to  $x$ , with wavenumber  $k$ , and Laplace transforming  
561 in time, with Laplace parameter  $s$ , the problem is reduced to ordinary differential equations

562 in  $y$ . Transformed fields are denoted as  $\hat{p}$  for  $p$ , etc. The transformed equations can be  
 563 solved analytically. Disregarding the solution terms that diverge at infinity, we find

$$\hat{\mathcal{E}}^\pm = C_1^\pm \exp(\pm|k|y) + C_2^\pm \exp(\pm\sqrt{k^2 + s/cy}), \quad (\text{A.3})$$

$$\hat{\mathcal{S}}^\pm = C_3^\pm \exp(\pm|k|y), \quad (\text{A.4})$$

564 where  $C_1^\pm$ ,  $C_2^\pm$ , and  $C_3^\pm$  are determined by interface conditions. Transforming the displace-  
 565 ments functions into physical fields give the displacements and pore pressure as [41]

$$\hat{u}_x^\pm = -ik\hat{\mathcal{E}}^\pm + ik y \hat{\mathcal{S}}^\pm, \quad (\text{A.5})$$

$$\hat{u}_y^\pm = -\frac{\partial \hat{\mathcal{E}}^\pm}{\partial y} + y \frac{\partial \hat{\mathcal{S}}^\pm}{\partial y} - (3 - 4\nu_u) \hat{\mathcal{S}}^\pm, \quad (\text{A.6})$$

$$\hat{p}^\pm = -G \frac{2(1 - \nu)}{\alpha(1 - 2\nu)} \left[ -k^2 \hat{\mathcal{E}}^\pm + \frac{\partial^2 \hat{\mathcal{E}}^\pm}{\partial y^2} - \frac{2(\nu_u - \nu)}{1 - \nu} \frac{\partial \hat{\mathcal{S}}^\pm}{\partial y} \right]. \quad (\text{A.7})$$

566 The stresses  $\sigma_{ij}$  are obtained from Hooke's law,

$$\sigma_{ij} = 2G\epsilon_{ij} + \frac{2G\nu}{1 - 2\nu} \epsilon_{kk} \delta_{ij} - \alpha p \delta_{ij}, \quad (\text{A.8})$$

567 where  $\delta_{ij}$  is the Kronecker delta and  $\epsilon_{ij}$  is the strain tensor, relevant transformed components  
 568 of which are written as  $\hat{\epsilon}_{xx}^\pm = ik\hat{u}_x^\pm$  and  $\hat{\epsilon}_{xy}^\pm = (ik\hat{u}_y^\pm + \hat{u}_{x,y}^\pm)/2$ , for example.

569 The  $C_1^\pm$ ,  $C_2^\pm$ , and  $C_3^\pm$  are then determined using a symbolic manipulator where the  
 570 appropriate boundary conditions are matched (Sections 2.2 and 2.3). Then the pore pressure  
 571 and shear stress are computed at the interface  $y \rightarrow 0^\pm$ , which finally grants expressions  
 572 presented in Section 3.1.

## 573 References

- 574 [1] I. Svetlizky, J. Fineberg, Classical shear cracks drive the onset of dry frictional motion, *Nature*  
 575 509 (7499) (2014) 205. doi:10.1038/nature13202.
- 576 [2] C. Marone, Laboratory-derived friction laws and their application to seismic faulting, *Annu. Rev. Earth*  
 577 *Pl. Sc.* 26 (1) (1998) 643–696.

- 578 [3] Z. Peng, J. Gomberg, An integrated perspective of the continuum between earthquakes and slow-slip  
579 phenomena, *Nat. Geosci.* 3 (9) (2010) 599. doi:10.1038/ngeo940.
- 580 [4] G. C. Beroza, S. Ide, Slow earthquakes and nonvolcanic tremor, *Annu. Rev. Earth Pl. Sc.* 39 (1) (2011)  
581 271–296. doi:10.1146/annurev-earth-040809-152531.
- 582 [5] M. D. Zoback, A. Kohli, I. Das, M. W. McClure, The importance of slow slip on faults during hydraulic  
583 fracturing stimulation of shale gas reservoirs, in: *SPE Americas Unconventional Resources Conference*,  
584 Society of Petroleum Engineers, 2012.
- 585 [6] Y. Guglielmi, F. Cappa, J.-P. Avouac, P. Henry, D. Elsworth, Seismicity triggered by fluid injection–  
586 induced aseismic slip, *Science* 348 (6240) (2015) 1224–1226. doi:10.1126/science.aab0476.
- 587 [7] J. Galeano, P. Espanol, M. Rubio, Experimental and theoretical results of stress relaxations in a model  
588 of earthquake dynamics, *Europhys. Lett.* 49 (4) (2000) 410.
- 589 [8] M. A. Rubio, J. Galeano, Stick-slip dynamics in the relaxation of stresses in a continuous elastic  
590 medium, *Phys. Rev. E* 50 (2) (1994) 1000.
- 591 [9] T. Baumberger, C. Caroli, O. Ronsin, Self-healing slip pulses along a gel/glass interface, *Phys. rev.*  
592 *lett.* 88 (7) (2002) 075509.
- 593 [10] T. Baumberger, C. Caroli, O. Ronsin, Self-healing slip pulses and the friction of gelatin gels, *Eur. Phys.*  
594 *J. E* 11 (1) (2003) 85–93.
- 595 [11] O. Ronsin, T. Baumberger, C. Hui, Nucleation and propagation of quasi-static interfacial slip pulses,  
596 *J. Adhesion* 87 (5) (2011) 504–529.
- 597 [12] N. Kato, A possible model for large preseismic slip on a deeper extension of a seismic rupture plane,  
598 *Earth Planet. Sc. Lett.* 216 (1-2) (2003) 17–25.
- 599 [13] A. M. Rubin, Episodic slow slip events and rate-and-state friction, *J. Geophys. Res. Solid Earth*  
600 113 (B11). doi:10.1029/2008JB005642.
- 601 [14] P. Segall, A. M. Rubin, A. M. Bradley, J. R. Rice, Dilatant strengthening as a mechanism for slow slip  
602 events, *J. Geophys. Res. Solid Earth* 115 (B12).
- 603 [15] Y. Bar-Sinai, E. A. Brener, E. Bouchbinder, Slow rupture of frictional interfaces, *Geophys. Res. Lett.*  
604 39 (3). doi:10.1029/2011GL050554.
- 605 [16] Y. Bar-Sinai, R. Spatschek, E. A. Brener, E. Bouchbinder, On the velocity-strengthening behavior of  
606 dry friction, *J. Geophys. Res. Solid Earth* 119 (3) (2014) 1738–1748. doi:10.1002/2013JB010586.
- 607 [17] A. H. Kohli, M. D. Zoback, Frictional properties of shale reservoir rocks, *J. Geophys. Res. Solid Earth*  
608 118 (9) (2013) 5109–5125. doi:10.1002/jgrb.50346.
- 609 [18] M. J. Ikari, C. Marone, D. M. Saffer, A. J. Kopf, Slip weakening as a mechanism for slow earthquakes,  
610 *Nat. Geosci.* 6 (6) (2013) 468. doi:10.1038/ngeo1818.
- 611 [19] D. M. Saffer, L. M. Wallace, The frictional, hydrologic, metamorphic and thermal habitat of shallow

- 612 slow earthquakes, *Nat. Geosci.* 8 (8) (2015) 594. doi:10.1038/ngeo2490.
- 613 [20] R. Bürgmann, The geophysics, geology and mechanics of slow fault slip, *Earth Planet. Sc. Lett.* 495  
614 (2018) 112–134.
- 615 [21] J. R. Rice, D. A. Simons, The stabilization of spreading shear faults by coupled deformation-  
616 diffusion effects in fluid-infiltrated porous materials, *J. Geophys. Res.* 81 (29) (1976) 5322–5334.  
617 doi:10.1029/JB081i029p05322.
- 618 [22] J. R. Rice, M. P. Cleary, Some basic stress diffusion solutions for fluid-saturated elastic porous media  
619 with compressible constituents, *Rev. Geophys.* 14 (2) (1976) 227–241. doi:10.1029/RG014i002p00227.
- 620 [23] J. W. Rudnicki, D. A. Koutsibelas, Steady propagation of plane strain shear cracks on an impermeable  
621 plane in an elastic diffusive solid, *Int. J. Solids Struct.* 27 (2) (1991) 205–225.
- 622 [24] J. Weertman, Unstable slippage across a fault that separates elastic media of different elastic constants,  
623 *J. Geophys. Res. Solid Earth* 85 (B3) (1980) 1455–1461.
- 624 [25] F. M. Chester, J. P. Evans, R. L. Biegel, Internal structure and weakening mechanisms of the san  
625 andreas fault, *J. Geophys. Res.* 98 (B1) (1993) 771–786.
- 626 [26] J. S. Caine, J. P. Evans, C. B. Forster, Fault zone architecture and permeability structure, *Geology*  
627 24 (11) (1996) 1025–1028.
- 628 [27] C. A. Wibberley, T. Shimamoto, Internal structure and permeability of major strike-slip fault zones:  
629 the Median Tectonic Line in Mie Prefecture, Southwest Japan, *J. Struct. Geol.* 25 (1) (2003) 59–78.
- 630 [28] J. Behnsen, D. Faulkner, Water and argon permeability of phyllosilicate powders under medium to  
631 high pressure, *Earth Planet. Sc. Lett.* 116 (B12).
- 632 [29] F. Chester, J. M. Logan, Implications for mechanical properties of brittle faults from observations of  
633 the punchbowl fault zone, california, *Pure Appl. Geophys.* 124 (1-2) (1986) 79–106.
- 634 [30] T. Mitchell, D. Faulkner, Towards quantifying the matrix permeability of fault damage zones in low  
635 porosity rocks, *Earth Planet. Sc. Lett.* 339 (2012) 24–31.
- 636 [31] F. M. Chester, J. S. Chester, Ultracataclasite structure and friction processes of the punchbowl  
637 fault, San Andreas system, California, *Tectonophysics* 295 (1) (1998) 199 – 221. doi:10.1016/S0040-  
638 1951(98)00121-8.
- 639 [32] F. M. Chester, J. S. Chester, D. L. Kirschner, S. E. Schulz, J. P. Evans, Structure of large-displacement,  
640 strike-slip fault zones in the brittle continental crust, *Rheology and deformation in the lithosphere at  
641 continental margins* 1 (2004) 223–260.
- 642 [33] O. Dor, T. K. Rockwell, Y. Ben-Zion, Geological observations of damage asymmetry in the structure of  
643 the San Jacinto, San Andreas and Punchbowl faults in Southern California: A possible indicator for pre-  
644 ferred rupture propagation direction, *Pure Appl. Geophys.* 163 (2) (2006) 301–349. doi:10.1007/s00024-  
645 005-0023-9.

- 646 [34] J. W. Rudnicki, J. R. Rice, Effective normal stress alteration due to pore pressure changes induced  
647 by dynamic slip propagation on a plane between dissimilar materials, *J. Geophys. Res. Solid Earth*  
648 111 (B10). doi:10.1029/2006JB004396.
- 649 [35] E. M. Dunham, J. R. Rice, Earthquake slip between dissimilar poroelastic materials, *J. Geophys. Res.*  
650 *Solid Earth* 113 (B9), b09304. doi:10.1029/2007JB005405.
- 651 [36] J. R. Rice, N. Lapusta, K. Ranjith, Rate and state dependent friction and the stability of sliding  
652 between elastically deformable solids, *J. Mech. Phys. Solids* 49 (9) (2001) 1865–1898.
- 653 [37] E. A. Brener, M. Weikamp, R. Spatschek, Y. Bar-Sinai, E. Bouchbinder, Dynamic instabilities of  
654 frictional sliding at a bimaterial interface, *J. Mech. Phys. Solids* 89 (2016) 149–173.
- 655 [38] M. Aldam, S. Xu, E. A. Brener, Y. Ben-Zion, E. Bouchbinder, Nonmonotonicity of the frictional  
656 bimaterial effect, *J. Geophys. Res. Solid Earth* 122 (10) (2017) 8270–8284.
- 657 [39] H. Shlomai, J. Fineberg, The structure of slip-pulses and supershear ruptures driving slip in bimaterial  
658 friction, *Nat. Commun.* 7 (2016) 11787.
- 659 [40] M. Aldam, Y. Bar-Sinai, I. Svetlizky, E. A. Brener, J. Fineberg, E. Bouchbinder, Frictional sliding with-  
660 out geometrical reflection symmetry, *Phys. Rev. X* 6 (2016) 041023. doi:10.1103/PhysRevX.6.041023.
- 661 [41] E. Detournay, A. H.-D. Cheng, Fundamentals of poroelasticity, in: *Analysis and Design Methods*,  
662 Elsevier, 1995, pp. 113–171.
- 663 [42] Y. Song, J. W. Rudnicki, Plane-strain shear dislocation on a leaky plane in a poroelastic solid, *J. Appl.*  
664 *Mech.* 84 (2) (2017) 021008.
- 665 [43] M. F. Linker, J. H. Dieterich, Effects of variable normal stress on rock friction: Observations and  
666 constitutive equations, *J. Geophys. Res. Solid Earth* 97 (B4) (1992) 4923–4940. doi:10.1029/92JB00017.
- 667 [44] M. Svärd, J. Nordström, Review of summation-by-parts-operators schemes for initial-boundary-value  
668 problems, *J. Comput. Phys.* 268 (0) (2014) 17–38.
- 669 [45] D. C. Del Rey Fernández, J. E. Hicken, D. W. Zingg, Review of summation-by-parts operators with  
670 simultaneous approximation terms for the numerical solution of partial differential equations, *Comput.*  
671 *Fluids* 95 (2014) 171 – 196.
- 672 [46] M. H. Carpenter, D. Gottlieb, S. Abarbanel, Time-stable boundary conditions for finite-difference  
673 schemes solving hyperbolic systems: Methodology and application to high-order compact schemes, *J.*  
674 *Comput. Phys.* 111(2) (1994) 220–236.
- 675 [47] K. Torberntsson, V. Stiernström, K. Mattsson, E. M. Dunham, A finite difference method for earth-  
676 quake sequences in poroelastic solids, *Computat. Geosci.* 22 (5) (2018) 1351–1370. doi:10.1007/s10596-  
677 018-9757-1.
- 678 [48] H. Gao, D. A. Schmidt, R. J. Weldon, Scaling relationships of source parameters for slow slip events,  
679 *Bull. Seismol. Soc. Am* 102 (1) (2012) 352–360.

- 680 [49] R. Righetti, J. Ophir, S. Srinivasan, T. A. Krouskop, The feasibility of using elastography for  
681 imaging the poisson's ratio in porous media, *Ultrasound Med. Biol.* 30 (2) (2004) 215 – 228.  
682 doi:10.1016/j.ultrasmedbio.2003.10.022.
- 683 [50] E. A. Brener, S. V. Malinin, V. I. Marchenko, Fracture and friction: Stick-slip motion, *Eur. Phys. J.*  
684 *E.* 17 (1) (2005) 101–113. doi:10.1140/epje/i2004-10112-3.
- 685 [51] J. P. Gong, Friction and lubrication of hydrogelsits richness and complexity, *Soft matter* 2 (7) (2006)  
686 544–552. doi:10.1039/B603209P.
- 687 [52] A. Ghosh, J. E. Vidale, J. R. Sweet, K. C. Creager, A. G. Wech, H. Houston, E. E. Brodsky, Rapid,  
688 continuous streaking of tremor in Cascadia, *Geochem. Geophys. Geosy.* 11 (12).
- 689 [53] Y. Peng, A. M. Rubin, High-resolution images of tremor migrations beneath the Olympic penin-  
690 sula from stacked array of arrays seismic data, *Geochem. Geophys. Geosy.* 17 (2) (2016) 587–601.  
691 doi:10.1002/2015GC006141.
- 692 [54] W. B. Frank, B. Rousset, C. Lasserre, M. Campillo, Revealing the cluster of slow transients behind a  
693 large slow slip event, *Sci. Adv.* 4 (5). doi:10.1126/sciadv.aat0661.
- 694 [55] D. R. Shelly, G. C. Beroza, S. Ide, Complex evolution of transient slip derived from precise tremor  
695 locations in western shikoku, japan, *Geochem. Geophys. Geosy.* 8 (10). doi:10.1029/2007GC001640.
- 696 [56] D. R. Shelly, Migrating tremors illuminate complex deformation beneath the seismogenic san andreas  
697 fault, *Nature* 463 (7281) (2010) 648.
- 698 [57] R. Nakata, N. Suda, H. Tsuruoka, Non-volcanic tremor resulting from the combined effect of Earth  
699 tides and slow slip events, *Nat, Geosci.* 1 (10) (2008) 676.
- 700 [58] J. W. Rudnicki, M. Wu, Mechanics of dip-slip faulting in an elastic half-space, *J. Geophys. Res. Solid*  
701 *Earth* 100 (B11) (1995) 22173–22186. doi:10.1029/95JB02246.
- 702 [59] Y. Fukao, Tsunami earthquakes and subduction processes near deep-sea trenches, *J. Geophys. Res.*  
703 *Solid Earth* 84 (B5) (1979) 2303–2314. doi:10.1029/JB084iB05p02303.
- 704 [60] D. A. Wiens, S. Anandakrishnan, J. P. Winberry, M. A. King, Simultaneous teleseismic and geodetic  
705 observations of the stick–slip motion of an Antarctic ice stream, *Nature* 453 (7196) (2008) 770.
- 706 [61] N. R. Iverson, Shear resistance and continuity of subglacial till: hydrology rules, *J. Glaciol.* 56 (200)  
707 (2010) 1104–1114.
- 708 [62] W. H. Schulz, J. P. McKenna, J. D. Kibler, G. Biavati, Relations between hydrology and velocity  
709 of a continuously moving landslide-Evidence of pore-pressure feedback regulating landslide motion?,  
710 *Landslides* 6 (3) (2009) 181–190.
- 711 [63] P. Bhattacharya, R. C. Viesca, Fluid-induced aseismic fault slip outpaces pore-fluid migration, *Science*  
712 364 (6439) (2019) 464–468. doi:10.1126/science.aaw7354.
- 713 [64] A. Verruijt, Displacement functions in the theory of consolidation or in thermoelasticity, *Zeitschrift für*

714 angewandte Mathematik und Physik ZAMP 22 (5) (1971) 891–898.

715 [65] J. McNamee, R. E. Gibson, Plane strain and axially symmetric problems of the consolidation of a  
716 semi-infinite clay stratum, Q. J. Mech. Appl. Math. 13 (2) (1960) 210–227.

717 [66] M. A. Biot, General solutions of the equations of elasticity and consolidation for a porous material, J.  
718 Appl. Mech. 23 (1) (1956) 91–96.